The surface temperature of a planet

Note that this derivation is for a rapidly rotating planet, in that the spin is fast enough that all of the planet recieves incomming radiation on the same timescale (or less) than it radiates it.

The power W (J s^{-1}) radiated per unit area by a blackbody is

$$W = \sigma T^4$$

with T in K, and σ is the Stefan-Boltzmann constant (5.67×10^{-8} W m^{-2} K^{-4}). Therefore the energy E radiated per unit time from a spherical blackbody or radius r is

$$E_{\rm rad} = 4\pi r^2 W$$

and the temperature of that blackbody is

$$T = \left(\frac{E_{\rm rad}}{4\pi\sigma r^2} \right)^{1/4}$$

The energy from a star of luminosity L at a distance d incident on a planet of radius r (the 'insolation') is

$$E_{\rm insol} = L \frac{\pi r^2}{4\pi d^2}$$

However, not all of the stellar energy that is incident with a planet reaches the planet's surface.

A fraction α (also often called A) of the energy is reflected directly back into space (mainly by clouds or ice). The fraction of the incident energy reflected is the *albedo* of the planet (formally their are two types of albedo, but we will ignore this technical point). Thee albedo of Earth is ~ 0.3, and of Venus ~ 0.7 (the large albedo of Venus is due to its complete cloud cover).

In addition, a fraction β of energy can be absorbed by the atmosphere (especially in the UV and IR) never reaching the ground (this is the reason for the temperature inversion at ~ 10 km at the top of the troposphere and the bottom of the stratosphere). We will ignore this factor, but see http://www.dangermouse.net/gurps/science/temps.html for a full derivation with this factor included if you really want to.

Thus the total energy that reaches the surface of a planet is

$$E_{\rm surf} = (1-\alpha) L \frac{\pi r^2}{4\pi d^2}$$

A planet can be assumed to be in energy balance - that is the amount of energy radiated by the planet equals the amount the planet absorbs, so that

$$E_{\rm surf} = E_{\rm rad}$$

If the planet is rapidly spinning, then we can assume that the whole surface of the planet is heated and re-radiates as a black body, so we can solve for the surface temperature:

$$T_{\rm surf} = \left(\frac{(1-\alpha)L}{16\pi\sigma d^2} \right)^{1/4}$$

Note that this is independant of the planet's radius.

Putting-in values for the Earth and Sun where $L_\odot = 3.83 \times 10^{26}$ W, $d = 1.5 \times 10^{11}$ m and $\alpha = 0.31$ gives a $T_{\rm surf} \sim 245$ K (note that if we'd done this including the effects of the atmospheric absorption of incomming radiation we would have got ~ 250 K).

But the average surface temperature of the Earth is ~ 286 K (13 C). This is because we have ignored the greenhouse effect - the trapping of heat radiated by the surface by the atmosphere. For the Earth the greenhouse effect increses the surface temperature by $35 - 40$ K!

The same calculation for Venus gives a surface temperature of ~ 256 K - only slightly warmer than the Earth. However, the surface temperature of Venus is actually ~ 735 K. A huge 490 K of greenhouse warming!

Note that these are *average* surface temperatures: latitude, geography, seasons, local albedo, local cloud cover etc. cause variations in the earth's surface temperature of ± 30 –40 K (compare Northern Canada with Arabia).

Another minor factor is heat released from inside a planet. The larger terrestrial planets have molten cores heated by radioactive decay. Heat is released in small amounts across the planet, but especially close to volcanoes, however this is normally a negligable amount. In extreme conditions - such as Io (and to a lesser extent Europa) - tidal forces from a massive nearby body can heat the interior of a planet/moon.

Retaining an atmosphere

The rms velocity of a gas particle of mass m at a temperature T is

$$v_{\rm rms} = \sqrt{\frac{3k_B T}{m}}$$

where k_B is Boltzmann's constant (1.38×10^{-23} m^2 kg s^{-2} K^{-1}). The escape velocity from a planet of radius r and mass M is

$$v_{\rm esc} = \sqrt{\frac{2GM}{r}}$$

where G is Newton's constant of gravitation (6.67×10^{-11} m^3 kg^{-1} s^{-2}).

In order to retain an atmosphere for a significant time we require $v_{\rm rms} < v_{\rm esc}/6$.

The factor of ~ 6 comes from a Maxwellian velocity distribution, one particle in 10^{16} has more than ~ 6 times the rms velocity, and as high velocity molecules are lost, the Maxwellian re-establishes itself leading to more particles above the escape velocity.

For the Earth $v_{\rm esc} = 11.2$ km s^{-1}. At a temperature of 286 K $v_{\rm rms}$ for H_2 is 1.9 km s^{-1}, whilst for O_2 it is 0.5 km s^{-1}. Thus for hydrogen the high-velocity tail of the Maxwellian that is $> v_{\rm esc}$ is sufficient for the loss of molecules into space.